Engineering Maths First Aid Kit

4.4

Trigonometrical Identities

Introduction

Very often it is necessary to rewrite expressions involving sines, cosines and tangents in alternative forms. To do this we use formulas known as **trigonometric identities**. A number of commonly used identities are listed here:

1. The identities

$$\tan A = \frac{\sin A}{\cos A} \qquad \sec A = \frac{1}{\cos A} \qquad \operatorname{cosec} A = \frac{1}{\sin A} \qquad \cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A, \quad \tan^2 A + 1 = \sec^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}, \qquad \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

Note: $\sin^2 A$ is the notation used for $(\sin A)^2$. Similarly $\cos^2 A$ means $(\cos A)^2$ and so on.