Trigonometrical Identities

Introduction

Very often it is necessary to rewrite expressions involving sines, cosines and tangents in alternative forms. To do this we use formulas known as trigonometric identities. A number of commonly used identities are listed here:

1. The identities

\[ \tan A = \frac{\sin A}{\cos A} \quad \sec A = \frac{1}{\cos A} \quad \cosec A = \frac{1}{\sin A} \quad \cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} \]

\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]

\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]

\[ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]

\[ 2 \sin A \cos B = \sin(A + B) + \sin(A - B) \]

\[ 2 \cos A \cos B = \cos(A - B) + \cos(A + B) \]

\[ 2 \sin A \sin B = \cos(A - B) - \cos(A + B) \]

\[ \sin^2 A + \cos^2 A = 1 \]

\[ 1 + \cot^2 A = \cosec^2 A, \quad \tan^2 A + 1 = \sec^2 A \]

\[ \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \]

\[ \sin 2A = 2 \sin A \cos A \]
\[
\sin^2 A = \frac{1 - \cos 2A}{2}, \quad \cos^2 A = \frac{1 + \cos 2A}{2}
\]
\[
\sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]
\[
\sin A - \sin B = 2 \cos \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right)
\]
\[
\cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]
\[
\cos A - \cos B = 2 \sin \left( \frac{A + B}{2} \right) \sin \left( \frac{B - A}{2} \right)
\]

Note: \( \sin^2 A \) is the notation used for \((\sin A)^2\). Similarly \( \cos^2 A \) means \((\cos A)^2\) and so on.