MATHEMATICS

SUPPORT CENTRE

Title: Simultaneous equations and the method of substitution.

Target: On completion of this worksheet you should be able to solve quadratic and linear simultaneous equations by substitution.

It is impossible to solve some pairs of equations by elimination.

<u>Example.</u>

$$3x + y = 11 \text{ and } x^2 + 2y^2 = 59.$$

We therefore have an alternative method- the **method of substitution.**

As with elimination the method of substitution reduces the two equations to one equation with one unknown.

To apply the method of substitution we should:

- Rearrange one equation to make one of the unknowns the subject.
- Substitute the expression for this unknown into the other equation.

<u>Example.</u>

Solve the simultaneous equations:

$$3r + 2s = 8 \text{ and } 2r - 4s = 16.$$

First name the equations to avoid confusion.

• $3r + 2s = 8$

, $2r - 4s = 16.$

Rearrange • to make r the subject.

$$3r + 2s = 8 \qquad [-2s]$$

$$\Rightarrow 3r = 8 - 2s \qquad [\div 3]$$

$$\Rightarrow r = \frac{8-2s}{3}.$$

Substitute into **,** . Use brackets to avoid mistakes.

$$2\left(\frac{8-2s}{3}\right) - 4s = 16.\ \mathbf{L}$$

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Then solve, expanding the brackets carefully!

$$\frac{16-4s}{3} - 4s = 16 \qquad [+4s]$$

$$\Rightarrow \frac{16-4s}{3} = 16 + 4s \qquad [\times 3]$$

$$\Rightarrow 16 - 4s = 48 + 12s \qquad [+4s]$$

$$\Rightarrow 16 = 48 + 16s \qquad [-48]$$

$$\Rightarrow -32 = 16s \qquad [\div 16]$$

$$\Rightarrow -2 = s.$$

Then we substitute this value of s into one of the original equations to obtain the full solution,

$$3r - 4 = 8.$$

Hence $r = 4$.

<u>Exercise.</u>

Solve the following pairs of simultaneous equations using the method of substitution.

1. $a + b = 7$, $\quad 2a + 3b = 18$.
2. $s + 2t = 14$, $\quad 3s - t = 0$.
3. $2a - 3b = 0$, $\quad 3a - 2b = 5$.
4. $4w - 3t = 34$, $\quad 2t + 3w = 17$.
5. $7p + 6 = 2r$, $\quad 3r - 2p = 26$.
6. $z + 2l - 13 = 7$, $\quad 3z - 2l = -4$.

(Answers: a=3, b=4; s= 2, t=6; a=3, b=2; w=7, t=-2; p=2, r=10; z=4, l=8.)

This method may appear to be more complicated than elimination but it enables us to solve non-linear simultaneous equations.

Whenever there are **indices** involved in simultaneous equations we should solve them by substitution. We must be careful when we expand our brackets. We should always write an expression out in full in order to avoid confusion.

When we solve a linear equation and a quadratic equation by substitution we should always first <u>rearrange the linear equation.</u>

<u>Example.</u>
Solve the following equations simultaneously.

$$x^2 + 3xy + y^2 = 31$$
$$2x - y = 1.$$

First name the equations.

• $\quad x^2 + 3xy + y^2 = 31$

, $\quad 2x - y = 1.$

Rearrange , to make y the subject.

$$y = 2x - 1. \qquad \textbf{f}$$

Substitute into • .

$$x^2 + 3x(2x-1) + (2x-1)^2.$$

Solve.

$$x^2 + 6x^2 - 3x + (2x-1)(2x-1) = 31$$
$$\Rightarrow 7x^2 - 3x + 4x^2 - 4x + 1 = 31$$
$$\Rightarrow 11x^2 - 7x + 1 = 31 \qquad [-31]$$
$$\Rightarrow 11x^2 - 7x - 30 = 0$$
$$\Rightarrow (11x+15)(x-2) = 0$$
$$\Rightarrow x = -\frac{15}{11},\ x = 2.$$

For each of these solutions we now need to find the value of *y*.

Substituting $x = -\frac{15}{11}$ into **f** gives

$$y = 2\times\left(-\frac{15}{11}\right) - 1 =$$
$$\Rightarrow y = -\frac{30}{11} - 1$$
$$\Rightarrow y = -\frac{41}{11}.$$

Similarly substituting $x = 2$ into **f** gives y = 3.
(Check this for yourself.)

<u>Exercise.</u>
Solve the following pairs of simultaneous equations.

1. $x - 2y = 1$
   $x^2 - 3xy + 2y^2 = 4.$

2. $3x + 2y = 8$
   $3x^2 - y^2 = 11.$

3. $x - y = 5$
   $2xy - y^2 = 56.$

4. $2x + 3y = 8$
   $x^2 - xy + y^2 = 3.$

(Answers: x=7, y=3; x=2, y=1 and x=-18, y=31;
x=9, y=4 and x=-9, y= -14;
x=1, y=2 and x=1.947, y=1.368.)