MATHEMATICS

SUPPORT CENTRE

Title: Changing the subject 1.

Target: On completion of this worksheet you should be able to change the subject of formulae including those with fractions, indices and roots.

An algebraic **formula** is an expression with an equals sign linking several variables.
E.g. $V=IR$.
The **subject** of a formula is the variable preceding the equal sign. In the previous example V was the subject.

It is often useful to **change the subject** of a formula. We should follow the same rule, do the same to both sides, as we follow with an equation. That is we can:

- Add the same amount to both sides.
- Subtract the same amount from both sides.
- Multiply both sides by the same amount.
- Divide both sides by the same amount.
- Square the whole of both sides.
- Square root the whole of both sides.

Examples.

1) Make d the subject of the formula $C = pd$.

$$C = pd \quad [\div p]$$
$$\Rightarrow \frac{C}{p} = d.$$

Remark: " Þ " is notation for implies.

2) Make h the subject of the formula $V = pr^2h$.

$$V = pr^2h \quad [\div pr^2]$$
$$\Rightarrow \frac{V}{pr^2} = h.$$

3) Make b the subject of the formula $a = \frac{b}{c}$.

$$a = \frac{b}{c} \quad [\times c]$$
$$\Rightarrow ac = b.$$

Exercise. Make the letter in brackets the subject.

1. $S = t + a$ (t)
2. $PV = T$ (V)
3. $2A = PQ$ (Q)
4. $V^2 = 4gh$ (h)
5. $a = \frac{p}{q}$ (p)
6. $v = u + at$ (u)

(Answers:
$t = S - a,\ V = \frac{T}{P},\ Q = \frac{2A}{P},\ h = \frac{V^2}{4g},\ p = aq, u = v - at.$)

Frequently more then one operation is required.
Examples.
Make t the subject of $v = u + at$.

$$v = u + at \quad [-u]$$
$$\Rightarrow v - u = at \quad [\div a]$$
$$\Rightarrow \frac{v-u}{a} = t.$$

Exercise. Make the letter in brackets the subject.

1) $H = S - dl$ (l)
2) $a = \frac{p}{q}$ (q)
3) $S = \frac{VAT}{500}$ (T)
4) $Y = \frac{Fl}{Ax}$ (x)
5) $I = \frac{E-p}{R+r}$ (r)

(Answers: $l = \frac{S-H}{d},\ q = \frac{p}{a},\ T = \frac{500S}{VA},$
$x = \frac{Fl}{YA},\ r = \frac{E-p-IR}{I}.$)

When the formula contains **brackets** we deal with it in exactly the same way. Sometimes it is useful to expand the brackets first.

Example.

Make x the subject in the formula $S = h(x-t)$.

$$S = hx - ht \quad [+ht]$$
$$\Rightarrow S + ht = hx \quad [\div h]$$
$$\Rightarrow \frac{S+ht}{h} = x.$$

Notice that we could have done this without expanding the brackets.

$$S = h(x-t) \quad [\div h]$$
$$\Rightarrow \frac{S}{h} = x - t \quad [+t]$$
$$\Rightarrow \frac{S}{h} + t = x.$$

Exercise. Make the letter in brackets the subject.

1) $R_2 = R_1(1-at)$ (t)
2) $2z = p(z-q)$ (q)
3) $T = \frac{2(S+P)}{Z}$ (S)
4) $T = \frac{l(x-C)}{C}$ (x)
5) $A = 2pr(r+h)$ (h)
6) $S = \frac{n}{2}\left(2a + (n-1)d\right)$ (d)

(Answers

$t = \frac{R_1 - R_2}{R_1 a}$, $q = \frac{pz - 2z}{p}$, $S = \frac{TZ}{2} - P$,

$x = \frac{TC}{l} + C$, $h = \frac{A}{2pr} - r$, $d = \frac{2S - 2an}{n(n-1)}$.)

When a formula contains **powers** or **roots** we must be careful about the order of the operations we use to transform the formulae.

In general rooting or taking powers should occur as late as possible to avoid algebraic mistakes.

Examples.

Make r the subject of the formulae

$a)\ V = \frac{4}{3}pr^3$, $\quad b)\ t = 2p\sqrt{\frac{l}{r}}$.

$$a)\ V = \frac{4}{3}pr^3 \quad [\times 3]$$
$$\Rightarrow 3V = 4pr^3 \quad [\div 4p]$$
$$\Rightarrow \frac{3V}{4p} = r^3 \quad [\sqrt[3]{\bullet}]$$
$$\Rightarrow \sqrt[3]{\frac{3V}{4p}} = r.$$

$$b)\ t = 2p\sqrt{\frac{r}{l}} \quad [\div 2p]$$
$$\Rightarrow \frac{t}{2p} = \sqrt{\frac{r}{l}} \quad [\bullet^2]$$
$$\Rightarrow \left(\frac{t}{2p}\right)^2 = \frac{r}{l} \quad [\times l]$$
$$\Rightarrow l\left(\frac{t}{2p}\right)^2 = r.$$

Exercise. Make the letter in brackets the subject.

1. $S = t^2$ (t)
2. $x = \sqrt{y}$ (y)
3. $s = t^2 + u$ (t)
4. $p = \sqrt{u} + as$ (u)
5. $V = \sqrt{gh}$ (h)
6. $v = k\sqrt{d}$ (d)
7. $t = 3h\sqrt{\frac{x}{y}}$ (x)
8. $R = PQ^2V$ (Q)
9. $\frac{x^2}{p^2} + \frac{y^2}{q^2} = 1$ (p)
10. $S = V + \frac{1}{2}at^2$ (t)

(Answers: $t = \sqrt{S}$, $y = x^2$, $t = \sqrt{s-u}$,

$u = (p-as)^2$, $h = \frac{V^2}{g}$, $d = \left(\frac{v}{k}\right)^2$, $x = y\left(\frac{t}{3h}\right)^2$,

$Q = \sqrt{\frac{R}{PV}}$, $p = \frac{xq}{\sqrt{q^2 - y^2}}$, $t = \sqrt{\frac{2(S-V)}{a}}$.))