Sometimes when we are changing the subject of a formula the variable that we want to make the subject occurs more than once. In this case it is necessary to transform the formula so that the variable is only written once. We can do this by

• Getting all the terms with the variable on one side of the formula.
• Taking the variable out as a common factor.

Example.
Make $x$ the subject of the formula

a) $cx + dx = m$ and b) $xy = x - bx + c$

\[
\begin{align*}
a) & \quad x(c + d) = m \quad [\div (c + d)] \\
& \Rightarrow x = \frac{m}{c + d}. \\
\end{align*}
\]

\[
\begin{align*}
b) & \quad xy = x - bx + c \quad [\div bx] \\
& \Rightarrow xy + bx = x + c \quad [-x] \\
& \Rightarrow xy + bx - x = c \\
& \Rightarrow x(y + b - 1) = c \quad [\div (y + b - 1)] \\
& \Rightarrow x = \frac{c}{y + b - 1}. \\
\end{align*}
\]

Exercise.
Make the letter in brackets the subject.

1. $by + ey = h$ \hspace{1cm} \text{(y)}
2. $ct + st - 3t = ms$ \hspace{1cm} \text{(t)}
3. $mnx + 3mx - 2x = 4p$ \hspace{1cm} \text{(x)}
4. $rt + ys = ft - gs$ \hspace{1cm} \text{(t)}
5. $3pq - 4p = tq - sq + 5$ \hspace{1cm} \text{(q)}

(Answers:
$3, 2, 3, 3, -4, 5, 4p$)

It often simplifies a problem involving fractions if we first transform the formula to one without fractions by multiplying the entire formula by the denominator of the fraction.

Example.
Make $x$ the subject of the formula $I = \frac{yx}{x + r}$.

\[
\begin{align*}
I &= \frac{yx}{x + r} \quad [\div (x + r)] \\
& \Rightarrow I(x + r) = yx \\
& \Rightarrow Ix + Ir = yx \quad [-Ix] \\
& \Rightarrow Ir = yx - Ix \\
& \Rightarrow Ir = x(y - I) \quad [\div (y - I)] \\
& \Rightarrow \frac{Ir}{y - I} = x.
\end{align*}
\]
Exercise.
Make the letter in brackets the subject.
1. \(mx = lx + p\) \((x)\)
2. \(m = \frac{uL}{L + rcR}\) \((L)\)
3. \(P = \frac{e^2m - e^2n}{S}\) \((e)\)
4. \(a + x = \frac{y}{2 + y}\) \((y)\)
5. \(\frac{1}{x} = \frac{4K - 2b}{3c + 5K}\) \((K)\)

(Answers: \(x = \frac{p}{m - l}, L = \frac{mrcR}{u - m}, e = \sqrt{\frac{Ps}{m - n}}, y = \frac{2(a + x)}{1 - a - x}, K = \frac{3c + 2bx}{4x - 5}\).)

When powers and roots are involved we should deal with them as late as possible.
Example.
Make \(p\) the subject of the formula
\[g = t \sqrt{\frac{4p + q}{gp - u}}.\]

\[g = t \sqrt{\frac{4p + q}{gp - u}} \quad [\div t]\]
\[\frac{g}{t} = \sqrt{\frac{4p + q}{gp - u}} \quad [\times (gp - u)]\]
\[(gp - u)\left(\frac{g}{t}\right)^2 = 4p + q\]
\[g\left(\frac{g}{t}\right)^2 - u\left(\frac{g}{t}\right)^2 = 4p + q \quad [-4p]\]
\[g\left(\frac{g}{t}\right)^2 - 4p - u\left(\frac{g}{t}\right)^2 = q \quad [+u\left(\frac{g}{t}\right)^2]\]
\[g\left(\frac{g}{t}\right)^2 - 4p = q + u\left(\frac{g}{t}\right)^2\]
\[p\left(\frac{g}{t}\right)^2 - 4 = q + u\left(\frac{g}{t}\right)^2 \quad [\div \left(\frac{g}{t}\right)^2 - 4]\]
\[q + u\left(\frac{g}{t}\right)^2\]
\[p = \frac{\left(\frac{g}{t}\right)^2}{\frac{g}{t} - 4}\]