Constant Acceleration Equations

For an object that has an initial velocity \( u \) and that is moving in a straight line with constant acceleration \( a \), the following equations connect the final velocity \( v \) and displacement \( s \) in a given time \( t \).

\[
\begin{align*}
  v &= u + at \\
  s &= \frac{1}{2}(u + v)t \\
  s &= ut + \frac{1}{2}at^2 \\
  s &= vt - \frac{1}{2}at^2 \\
  v^2 &= u^2 + 2as
\end{align*}
\]

Note: These equations cannot be used if the acceleration is not constant.

**Worked Example 1.**

A motorbike joins a motorway traveling at 10 m s\(^{-1}\), and increases speed to 30 m s\(^{-1}\) with a constant acceleration of 1.25 m s\(^{-2}\) along the straight road. How much time does this take, and how far does the bike travel in this time?

**Solution.**

Firstly consider what information has been given, namely \( u = 10 \) m s\(^{-1}\), \( v = 30 \) m s\(^{-1}\) and \( a = 1.25 \) m s\(^{-2}\).

The question asks for the values of \( t \) and then \( s \).

The equation that connects \( u, v, a \) and \( t \) is (1). Inserting the known values into (1) gives:

\[
\begin{align*}
  30 &= 10 + 1.25t \\
  20 &= 1.25t \\
  \Rightarrow t &= 16 \text{ s}
\end{align*}
\]

Now either equation (2), (3), (4) or (5) can be used to calculate \( s \). For example, using (2):

\[
s = \frac{1}{2}(u + v)t = \frac{1}{2}(10 + 30) \times 16 = 320 \text{ m}.
\]

**Worked Example 2.**

The driver of a car traveling along a straight road sees that the traffic lights, 40 metres away, have turned to red. Given that after 4 seconds the car stops exactly at the traffic lights, what is the deceleration of the car?

**Solution.**

Again, consider what information has been given, namely \( s = 40 \) m and \( t = 4 \) s.
It can also be deduced that because the car was at rest when it reached the traffic lights, \(v = 0 \text{ m s}^{-1}\). The question asks for the deceleration and so involves \(a\).

The equation that connects \(s, t, v\) and \(a\) is (4). Inserting the known values into (4) gives:

\[
\begin{align*}
\frac{s}{40} &= v t - \frac{1}{2} a t^2 \\
40 &= 0 \times 4 - \frac{1}{2} \times a \times 4^2 \\
40 &= -8a \\
\Rightarrow a &= -5.0 \text{ m s}^{-2} \text{ (to 2 s.f.)}
\end{align*}
\]

Therefore, the car decelerates at a rate of 5 m s\(^{-2}\)

**Worked Example 3.**

A child throws a tennis ball vertically upwards at 7.7 m s\(^{-1}\) from ground level. Assuming that no resistance forces act on the ball, so that it moves only under the influence of gravity (\(g = 9.81 \text{ m s}^{-2}\)), what is the maximum height the tennis ball reaches?

**Solution.**

Here, consider what information is already known and what can be used. It is known that \(u = 7.7 \text{ m s}^{-1}\) and \(a = -9.81 \text{ m s}^{-2}\) as gravity acts downwards and the positive direction is upwards. It can also be deduced that at the maximum height \(v = 0 \text{ m s}^{-1}\).

Therefore, using (5):

\[
\begin{align*}
v^2 &= u^2 + 2as \\
0 &= 7.7^2 + 2 \times (-9.81) \times s \\
0 &= 59.29 - 19.62 \times s \\
\Rightarrow s &= 3.0 \text{ m (to 2 s.f.)}
\end{align*}
\]

**Exercises**

1. A rally car accelerates from 10 m s\(^{-1}\) to 58 m s\(^{-1}\) in 8 seconds as it moves along a straight road. Given that the acceleration is constant, what is the acceleration of the car?
2. A bus traveling along a straight road accelerates at 2 m s\(^{-2}\), for 4 seconds, covering a distance of 44 metres. After the 4 seconds what velocity is the bus traveling at?
3. A rowing boat crosses the finish line at 12 m s\(^{-1}\) and carries on in a straight line. If it immediately decelerates at 4 m s\(^{-2}\) until it comes to rest, how far past the finish line will the rowing boat come to a stop?
4. During the middle of an 800 metre race an athlete running at 6.8 m s\(^{-1}\) constantly accelerates, along part of the straight, to 8 m s\(^{-1}\) in order to get in a better position for the final lap. Given this took 2 seconds, what distance did the athlete cover in this time?
5. A train leaves a station from rest and travels along a straight track. If after 20 seconds the train is 500 metres from the station, what is the acceleration of the train?
6. A lift at the ground floor rises vertically from rest with constant a acceleration of 0.6 m s\(^{-2}\). If it passes the first floor at 1.8 m s\(^{-1}\), how high is the first floor?

**Answers (all to 2 s.f.)**

1. 6 m s\(^{-2}\)  
2. 15 m s\(^{-1}\)  
3. 18 m  
4. 15 m  
5. 2.5 m s\(^{-2}\)  
6. 2.7 m