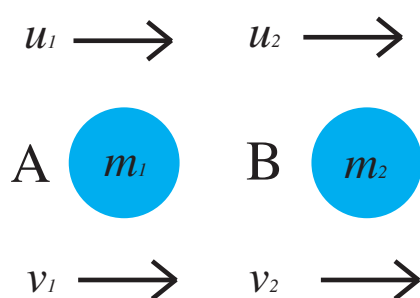


Conservation of Momentum

In this leaflet the concept of **conservation of momentum** will be introduced in both one and two dimensions.

Conservation of momentum

When no resultant external force acts on a system of interacting particles the total momentum of the system remains constant. Suppose two particles collide.



Given,

- m_1 = mass of particle A
- m_2 = mass of particle B
- u_1 = velocity of particle A before collision
- u_2 = velocity of particle B before collision
- v_1 = velocity of particle A after collision
- v_2 = velocity of particle B after collision

Then, the principle of conservation of momentum is:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Total momentum before collision = Total momentum after collision

Worked Example 1.

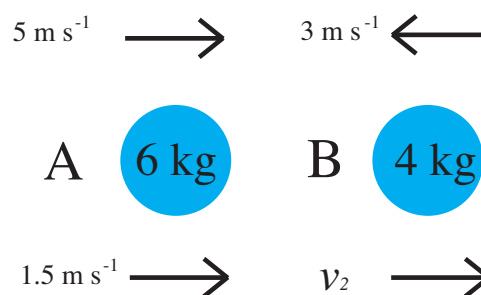
A particle A, of mass 6 kg, travelling in a straight line at 5 m s^{-1} collides with a particle B, of mass 4 kg, travelling in the same straight line, but in the opposite direction, with a speed of 3 m s^{-1} . Given that after the collision particle A continues to move in the same direction at 1.5 m s^{-1} , what speed does particle B move with after the collision?

Solution

It is always useful to depict the collision with the velocities both before and after.

Using the principle of conservation of momentum:

$$\begin{aligned} m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2 v_2 \\ 6 \times 5 + 4 \times (-3) &= 6 \times 1.5 + 4 \times v_2 \\ 9 &= 4 \times v_2 \\ v_2 &= \frac{9}{4} = 2.3 \text{ m s}^{-1} \quad (2 \text{ s.f.}) \end{aligned}$$



So after the collision particle B moves with a speed of 2.3 m s^{-1} , in the same direction as A.

Conservation of momentum in two dimensions

The principle of conservation of momentum can be used in two dimensions by expressing the velocities in vector form to obtain: $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$.

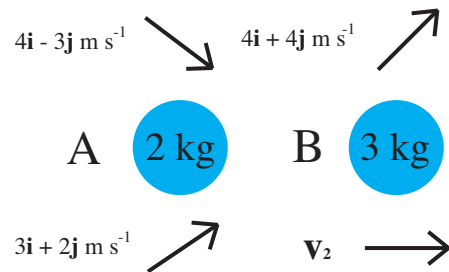
Worked Example 2.

A particle A, of mass 2 kg, collides with a particle B, of mass 3 kg. The velocity of particle A before the collision was $(4\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$ and the velocity of particle B before the collision was $(4\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Given the velocity of particle A after the collision was $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$, what was the velocity of B after the collision?

Solution

Here we have, $m_1 = 2$, $m_2 = 3$, $\mathbf{u}_1 = (4\mathbf{i} - 3\mathbf{j})$, $\mathbf{u}_2 = (4\mathbf{i} + 4\mathbf{j})$ and $\mathbf{v}_1 = (3\mathbf{i} + 2\mathbf{j})$

$$\begin{aligned} m_1\mathbf{u}_1 + m_2\mathbf{u}_2 &= m_1\mathbf{v}_1 + m_2\mathbf{v}_2 \\ 2(4\mathbf{i} - 3\mathbf{j}) + 3(4\mathbf{i} + 4\mathbf{j}) &= 2(3\mathbf{i} + 2\mathbf{j}) + 3\mathbf{v}_2 \\ 8\mathbf{i} - 6\mathbf{j} + 12\mathbf{i} + 12\mathbf{j} &= 6\mathbf{i} + 4\mathbf{j} + 3\mathbf{v}_2 \\ 14\mathbf{i} + 2\mathbf{j} &= 3\mathbf{v}_2 \\ \mathbf{v}_2 &= \frac{14}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} \\ &= 4.7\mathbf{i} + 0.67\mathbf{j} \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$



Exercises

1. A particle A, of mass 10 kg, travelling in a straight line at 4 m s^{-1} collides with a stationary particle B, of mass 12 kg. Given that after the collision particle A becomes stationary, what speed does particle B move with?
2. A particle A, of mass m_1 kg, travelling in a straight line at 6 m s^{-1} collides with a particle B, of mass 9 kg, which is travelling in the same direction, with a speed of 2 m s^{-1} . Given that after the collision particle A continues to move in the same direction at 1.5 m s^{-1} and particle B moves in the same direction as particle A at 5 m s^{-1} , what is the mass of particle A?
3. A particle A, of mass 2 kg, collides with a particle B, of mass 2 kg. The velocity of particle A before the collision was $(5\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ and the velocity of particle B before the collision was $7\mathbf{i} \text{ m s}^{-1}$. Given the velocity of particle B after the collision was $(6\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$, what was the velocity of particle A after the collision?
4. A particle A, of mass 8 kg, collides with a particle B, of mass m_2 kg. The velocity of particle A before the collision was $(-1\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ and the velocity of particle B before the collision was $(-0.8\mathbf{i} + 1.4\mathbf{j}) \text{ m s}^{-1}$. Given the velocity of particle A after the collision was $(-2\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$, and the velocity of particle B was $3\mathbf{j} \text{ m s}^{-1}$, what was the mass of particle B?

Answers (All to 2 s.f.)

1. 3.3 m s^{-1}
2. 6 kg
3. $6\mathbf{i} \text{ m s}^{-1}$
4. 10 kg