Logarithms

Introduction

We use logarithms to write expressions involving powers in a different form. If you can work confidently with powers you should have no problems handling logarithms.

Logarithms

Consider the statement

$$100 = 10^2$$

In this statement we say that 10 is the **base** and 2 is the **power** or **index**.

Logarithms are simply an alternative way of writing a statement such as this. We rewrite it as

$$\log_{10} 100 = 2$$

This is read as 'log to the base 10 of 100 is 2'.

As another example, since

$$2^5 = 32$$

we can write

$$\log_2 32 = 5$$

More generally,

$$\text{if} \qquad a = b^c, \qquad \text{then} \qquad \log_b a = c$$

The only restriction that is placed on the value of the base is that it is a positive real number excluding the number 1. In practice logarithms are calculated using only a few common bases. Most frequently you will meet bases 10 and e. The letter e stands for the number 2.718... and is used because it is found to occur in the mathematical description of some economic phenomena. Your calculator will be able to calculate logarithms to bases 10 and e. Usually the 'log' button is used for base 10, and the 'ln' button is used for base e. ('ln' stands for 'natural logarithm'). Check that you can use your calculator correctly by verifying that, correct to four decimal places,

$$\log_{10} 73 = 1.8633, \qquad \text{and} \qquad \log_e 5.64 = 1.7299$$

Exercises

1. Change the following log forms to the equivalent power forms.

(a) $\log_{10} 1000$ (b) $\log_2 8$ (c) $\log_3 9$ (d) $\ln e$ (e) $\log_{10} 1$

2. Change the following power forms to the equivalent log forms.

(a) $2^6 = 64$ (b) $10^3 = 1000$ (c) $81 = 9^2$ (d) $625 = 5^4$ (e) $7^0 = 1$

Answers

1. (a) 3 (b) 3 (c) 2 (d) 1 (e) 0
2. (a) $\log_2 64 = 6$ (b) $\log_{10} 1000 = 3$ (c) $\log_9 81 = 2$ (d) $\log_5 625 = 4$ (e) $\log_7 1 = 0$

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