

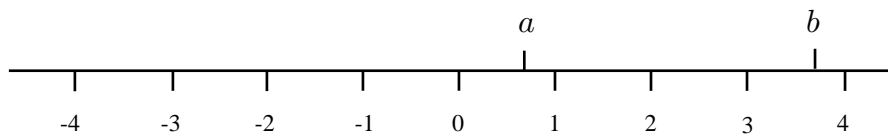
# Manipulating inequalities

## Introduction

This leaflet explains the meaning of the inequality symbols  $<$  and  $>$ , and shows how expressions involving them are manipulated.

## The number line and inequality symbols

A useful way of picturing numbers is to use a **number line**. The figure shows part of this line. Positive numbers are on the right-hand side of this line; negative numbers are on the left.



Numbers can be represented on a number line. If  $a < b$  then equivalently,  $b > a$ .

The symbol  $>$  means ‘greater than’; for example, since 6 is greater than 4 we can write  $6 > 4$ . Given any number, all numbers to the right of it on the line are greater than the given number. The symbol  $<$  means ‘less than’; for example, because  $-3$  is less than 19 we can write  $-3 < 19$ . Given any number, all numbers to the left of it on the line are less than the given number.

For any numbers  $a$  and  $b$ , note that if  $a$  is less than  $b$ , then  $b$  is greater than  $a$ . So the following two statements are equivalent:  $a < b$  and  $b > a$ . So, for example, we can write  $4 < 17$  in the equivalent form  $17 > 4$ .

If  $a < b$  and  $b < c$  we can write this concisely as  $a < b < c$ . Similarly if  $a$  and  $b$  are both positive, with  $b$  greater than  $a$  we can write  $0 < a < b$ .

## Rules for manipulating inequalities

To change or rearrange statements involving inequalities the following rules should be followed:

**Rule 1.** Adding or subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.

**Rule 2.** Multiplying or dividing both sides by a **positive** number leaves the inequality symbol unchanged.

**Rule 3.** Multiplying or dividing both sides by a **negative** number **reverses the inequality**. This means  $<$  changes to  $>$ , and vice versa.

So,

$$\text{if } a < b \text{ then } a + c < b + c \quad \text{using Rule 1}$$

For example, given that  $5 < 7$ , we could add 3 to both sides to obtain  $8 < 10$  which is still true.

Also, using Rule 2,

if  $a < b$  and  $k$  is positive, then  $ka < kb$

For example, given that  $5 < 8$  we can multiply both sides by 6 to obtain  $30 < 48$  which is still true.

Using Rule 3

if  $a < b$  and  $k$  is negative, then  $ka > kb$

For example, given  $5 < 8$  we can multiply both sides by  $-6$  and reverse the inequality to obtain  $-30 > -48$ , which is a true statement. A common mistake is to forget to reverse the inequality when multiplying or dividing by negative numbers.

## Solving inequalities

An inequality will often contain an unknown variable,  $x$ , say. To **solve** means to find all values of  $x$  for which the inequality is true. Usually the answer will be a range of values of  $x$ .

### Example

Solve the inequality  $7x - 2 > 0$ .

### Solution

We make use of the Rules to obtain  $x$  on its own. Adding 2 to both sides gives

$$7x > 2$$

Dividing both sides by the positive number 7 gives

$$x > \frac{2}{7}$$

Hence all values of  $x$  greater than  $\frac{2}{7}$  satisfy  $7x - 2 > 0$ .

### Example

Find the range of values of  $x$  satisfying  $x - 3 < 2x + 5$ .

### Solution

There are many ways of arriving at the correct answer. For example, adding 3 to both sides:

$$x < 2x + 8$$

Subtracting  $2x$  from both sides gives

$$-x < 8$$

Multiplying both sides by  $-1$  and **reversing the inequality** gives  $x > -8$ .

## Exercises

In each case solve the given inequality.

- $2x > 9$ ,
- $x + 5 > 13$ ,
- $-3x < 4$ ,
- $7x + 11 > 2x + 5$ ,
- $2(x + 3) < x + 1$

## Answers

- $x > 9/2$ ,
- $x > 8$ ,
- $x > -4/3$ ,
- $x > -6/5$ ,
- $x < -5$ .